

Can these problems be solved without formal algebra? Yes, indeed! There are strategies that include visual modeling and/or straightforward computation.

Problem #1:

Mervin had some cartons of milk. He sold $\frac{2}{5}$ of the cartons in the morning. He then sold $\frac{3}{4}$ of the remainder in the afternoon. He sold 24 more cartons in the afternoon than in the morning. How many cartons of milk did Mervin have at first?

One way to solve this is to use computation. Subtraction tells us that he still had $\frac{3}{5}$ of his cartons left at the start of the afternoon. Then he sold $\frac{3}{4}$ of $\frac{3}{5}$ of his original number.

$\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$ of his original number.

In the morning, he sold $\frac{2}{5}$ of his original number.

A common denominator would be helpful now, so let's change the $\frac{2}{5}$ into $\frac{8}{20}$.

So he sold $\frac{8}{20}$ of his cartons in the morning and $\frac{9}{20}$ in the afternoon.

He sold $\frac{1}{20}$ more in the afternoon than in the morning.

He sold 24 more in the afternoon, so that must be the value of the $\frac{1}{20}$.

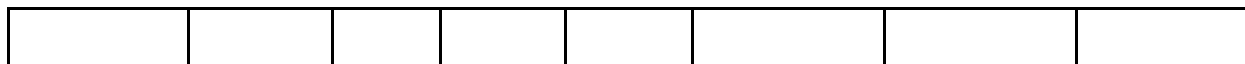
If $\frac{1}{20} = 24$, $\frac{20}{20} = 24 \times 20$, or 480.

That was his original supply — 480 cartons of milk.

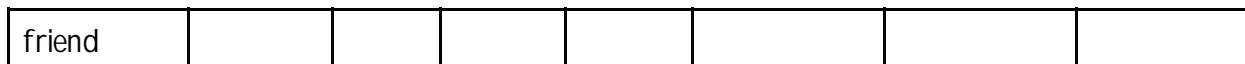
Problem #2:

Alice baked a certain number of pies. She gave $\frac{1}{8}$ of the pies to her friends and $\frac{1}{4}$ of the remainder to her neighbor. She was left with 63 pies. How many did Alice bake?

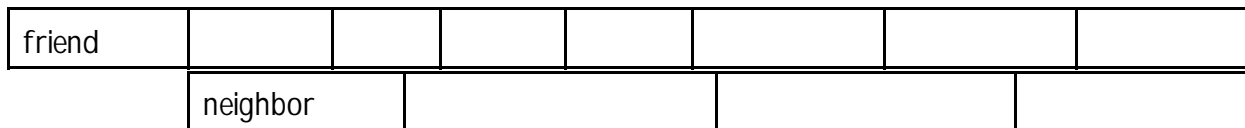
So let's draw a diagram — a long bar that represents the pies at the beginning — and cut it into eighths. (Students don't need to make these 8 parts identical in terms of measurement. They just need to know that the drawing is a *representation* of eighths.)



And we'll label the $\frac{1}{8}$ of her pies that she gave to her friend.:



Now we'll draw a new bar under the $\frac{7}{8}$ that Alice still has. We'll cut it into 4 parts and label one with the pies she gave her neighbor. That is, she gave away $\frac{1}{4}$ of $\frac{7}{8}$



She now has $\frac{3}{4}$ of $\frac{7}{8}$ of her pies left.

She has 63 pies left.

So each of those parts must represent 21 pies ($63 \div 3 = 21$).

friend							
	neighbor	21	21	21			

Since each of those fourths is worth 21, she must have given 21 pies to her neighbor:

friend							
	neighbor (21)	21	21	21			

All those fourths in the second bar add up to 84.

They represent $\frac{7}{8}$ of the original pies.

The friend got the other $\frac{1}{8}$ of the pies.

If $\frac{7}{8} = 84$, then $\frac{1}{8}$ must be worth 14.

The total number of pies = 14 plus 84, or 98 pies.

As you can see, the computations weren't difficult, but the problems had to be analyzed and represented correctly. And high school algebra was not required.